1) If \( f(x) = 1 - 3x - 2x^2 \) and \( g(x) = 3 - x \), find \((f \circ g)(x)\) and \((f \circ g)(-2)\).

2) If \( f(x) = \frac{x - 3}{2x + 5} \), find \( f^{-1}(x) \).

3) Use the definition of logarithms to rewrite this equation in exponential form: \( \log_5 A = 2 \).

4) Expand \( \log_5 \left( \frac{x^2 y^3}{z^4 \sqrt{w}} \right) \).

5) a) Evaluate \( \log_8 64 \) and show how you got the answer without a calculator.
   b) Evaluate to 3 decimal places: \( \log_5 632 \).

6) Solve: \( 32^x = 16^{1-x} \).

7) Solve: \( 3^{4-x} = 4^{2x-1} \). Round your answer to three decimal places.

8) Solve: \( \log x + \log(x + 48) = 2 \).

9) Solve: \( (3k - 1)(k + 4) = 11 \).

10) Solve: \( 2t^2 - t + 5 = 0 \).

11 and 12: No supposed to be included! (Copy/Paste error from Math 90)

11) Write the solution set in interval notation: \( x^2 - 5x - 6 \geq 0 \).

A) The amount of carbon-14 in the general environment causes 15 dpm (disintegrations per minute) per gram of total carbon. A wooden spoon from an archeological dig site shows 6 dpm per gram. How old is the spoon, to the nearest hundred years? (Assume carbon-14 has a half-life of 5700 years.)

B) Determine the half-life of a radioactive substance if a sample started with 8.4 milligrams 35 days ago, and it is now down to only 7.9 milligrams. Round to the nearest day.

C) Iodine-131 has a half-life of 8 days. A sealed room has 38 grams of iodine-131 in it. How much iodine-131 was in the room when it was sealed two weeks ago, to the nearest gram?

D) A farmer wants to build a rectangular pigpen next to her barn. She's got 90 feet of fencing available to use, and since the pigpen will be next to the barn, she only needs to fence three sides of the rectangle. What dimensions will give her the maximum possible area?

E) When a small business sells its product for \( x \) dollars, they bring in revenue of \( R = -0.3x^2 + 12x \) dollars per week. What price should they sell their product at to get the maximum possible revenue, and what will that maximum revenue be? If necessary, round to the nearest penny.

F) When a television is described as being a “53-inch Television”, that’s the diagonal measurement from corner to corner. If the width is 11 inches less than twice the height, what are the width and height of such a television?

G) An orange grower finds that she gets an average yield of 40 bushels per tree when she plants 20 trees on an acre of ground. Each time she adds one tree per acre, the yield per tree decreases by 3 bushels, due to congestion. How many trees per acre should she plant for maximum yield?

w) Graph the function \( g(x) = \log_3 (2 - x) \).

x) Graph the function \( k(x) = 2 + \log_{0.5} (x + 1) \).

y) Graph the function \( f(x) = 2^{1-|x|} \).

z) Graph the function \( h(x) = (\frac{1}{3})^{x^2 - 1} \).
1) If \( f(x) = 1 - 3x - 2x^2 \) and \( g(x) = 3 - x \), find \((f \circ g)(x)\) and \((f \circ g)(-2)\)

\[
(f \circ g)(x) = f(g(x)) = 1 - 3(g(x)) - 2(g(x))^2
\]
\[
= 1 - 3(3 - x) - 2(3 - x)^2
\]
\[
= 1 - 9 + 3x - 2(9 - 6x + x^2)
\]
\[
= 1 - 9 + 3x - 18 + 12x - 2x^2
\]
\[
= -2x^2 + 15x - 26
\]

\[(f \circ g)(-2) = f(g(-2)) = f(5) = 1 - 3 \cdot 5 - 2 \cdot 5^2 = -64\]

B) Determine the half-life of a radioactive substance if a sample started with 8.4 milligrams 35 days ago, and it is now down to only 7.9 milligrams. Round to the nearest day.

\[
P_0 e^{kt} \quad \text{Find } k : \quad 7.9 = 8.4 e^{35k}
\]
\[
\frac{7.9}{8.4} = e^{35k}
\]
\[
\ln\left(\frac{7.9}{8.4}\right) = 35k
\]
\[
\ln\left(\frac{7.9}{8.4}\right) \div 35 = k
\]

\[
\text{Answer the question: } \quad P = P_0 e^{kt}
\]
\[
4.2 = 8.4 e^{kt}
\]
\[
\frac{1}{2} = e^{kt}
\]
\[
\ln\left(\frac{1}{2}\right) = k \cdot t
\]
\[
\frac{\ln\left(\frac{1}{2}\right)}{k} = t
\]

\[
395, 316406\ldots = t
\]
7) Solve: $3^{4-x} = 4^{2x-1}$ Round your answer to three decimal places

\[
\log 3^{4-x} = \log 4^{2x-1} \\
(4-x) \log 3 = (2x-1) \log 4 \\
4 \log 3 - x \log 3 = 2x \log 4 - \log 4 + \log 3 + \log 3 + \log 3 \\
4 \log 3 + \log 4 = 2x \log 4 + x \log 3 \\
\frac{4 \log 3 + \log 4}{2 \log 4 + \log 3} = x \\
1.493268755 = x \\
1.493 = x
\]

6) Solve: $32^x = 16^{1-x}$

\[
5^x = (2^4)^{1-x} \\
25^x = 2^{4-4x} \\
5^x = 2^{4-4x} \\
a \cdot x = 4 \\
\frac{4}{a} = x
\]
4) Expand \( \log_5 \left( \frac{x^2y^3}{2^4\sqrt{w}} \right) = 2\log_5 x + 3\log_5 y - 4\log_5 2 - \frac{1}{2}\log_5 w \)

5) a) Evaluate \( \log_8 16 \) and show how you got the answer without a calculator.

b) Evaluate to 3 decimal places: \( \log_5 632 \)

\[
\begin{align*}
\text{a)} & \quad \log_8 16 = \, ? \\
& \quad 16 = 8^2 \quad \Rightarrow \quad 4 = 3 \cdot ? \\
& \quad 2^4 = 2^2 \quad \Rightarrow \quad \log_8 16 = \frac{4}{3} \\
\text{b)} & \quad \log_5 632 = \frac{\log_{10} 632}{\log_{10} 5} \\
& \quad 4.00\overline{6}92022\ldots \quad \Rightarrow \quad 4.007
\end{align*}
\]

8) Solve: \( \log x + \log (x + 48) = 2 \)

\[
\begin{align*}
\text{a)} & \quad \log (x(x + 48)) = 2 \\
& \quad x(x + 48) = 10^2 \\
& \quad x^2 + 48x = 100 \\
& \quad x^2 + 48x - 100 = 0 \\
& \quad (x + 50)(x - 2) = 0 \\
& \quad x = -50 \quad \checkmark \quad x = 2
\end{align*}
\]

\( \text{You MUST CHECK!} \)

- log (-50) + ...
  - Can't do a log of a negative!
- log (2) + log (2+48)
- \( \log 2 + \log 50 \)
- calc: 2! Yes!

Do NO ALGEBRA during the check!
3) Use the definition of logarithms to rewrite this equation in exponential form: \( \log_k A = 2 \)
2) If \( f(x) = \frac{x-3}{2x+5} \), find \( f^{-1}(x) \)

\[
y = \frac{x-3}{2x+5} \quad \text{Swap} \quad \frac{\chi}{2y+5} \quad \chi \text{ and } y
\]

\[
\chi = \frac{y-3}{2y+5} \\
\chi(2y+5) = y-3 \\
2\chi y + 5\chi = y-3 \\
2\chi y - y = -5\chi - 3 \\
\chi(2\chi-1) = -5\chi - 3 \\
\chi = \frac{-5\chi - 3}{2\chi - 1}
\]

\[
f^{-1}(\chi) = \frac{-5\chi - 3}{2\chi - 1}
\]

10) Solve: \( 2t^2 - t + 5 = 0 \)

\[
t = \frac{1 \pm \sqrt{1 - 4 \times 2 \times 5}}{4} = \frac{1 \pm \sqrt{-39}}{4} = \frac{1 \pm i \sqrt{39}}{4}
\]

\[
t = \frac{1}{4} \pm \frac{\sqrt{39}}{4} i
\]

\(- a + bi \text{ form for complex numbers} \)
9) Solve: \((3k - 1)(k + 4) = 11\)

\[
3k^2 + 11k - 4 = 11
\]

\[
3k^2 + 11k - 15 = 0
\]

\[
k = \frac{-11 \pm \sqrt{121 - 4 \cdot 3 \cdot (-15)}}{6}
\]

\[
k = \frac{-11 \pm \sqrt{301}}{6}
\]

E) When a small business sells its product for \(x\) dollars, they bring in revenue of \(R = -3x^2 + 12x\) dollars per week. What price should they sell their product at to get the maximum possible revenue, and what will that maximum revenue be? If necessary, round to the nearest penny.

Vertex: \(x = \frac{-b}{2a} = \frac{-12}{-6} = \frac{120}{6} = 20\)

Sell it for $20

The revenue will be $120 per week

\(R = -3 \cdot (20)^2 + 12 \cdot (20) = -120\)