Chapter 5 - Money!

Interest: Using money to make money

Simple Interest Formula: \( I = Prt \)

Example: An $8000 certificate earns 6.5% simple interest. How much will it be worth in 15 years?

\[
I = 8000 \cdot 0.065 \cdot 15 = 7800
\]

Total = 8000 + 7800 = 15,800

Future Value Formula (Simple Interest):

\[
FV = P + I = P(1 + rt)
\]
5.2 **Compound** Interest:

... at 8% per year, compounded monthly, for 5 years...

Key concepts: 
- \( i \) = interest rate per compounding period (not per year)
- \( n \) = total number of compounding periods (not years)

\[
\begin{align*}
\hat{i} &= \frac{0.08}{12} \\
\hat{n} &= 12 \cdot 5 = 60
\end{align*}
\]

So, the future value after 1 compounding period would be: 

\[
FV = P(1 + i)
\]

Then, that value becomes the new principal for the next compounding:

\[
FV = P(1 + i)(1 + i)
\]

And THAT becomes the new principal for the NEXT: 

\[
FV = P(1 + i)(1 + i)(1 + i)
\]

... for however many compoundings there are

So, the formula is: 

\[
FV = P \left(1 + \frac{i}{n}\right)^n
\]

Common types of compounding: Yearly, Quarterly, Monthly, Weekly

... at 8% per year, compounded monthly, for 5 years...

Let's compare retirement accounts. Both are $10,000. Both earn 8% yearly interest. Both will be kept for 20 years. One uses simple interest. The other uses monthly compounding. Compare the future values.

\[
\begin{align*}
FV_{\text{Simple Interest}} &= 10000 \left(1 + \frac{0.08}{12}\right)^{240} \\
&= \$49,268.03 \\

FV_{\text{Monthly Compounding}} &= 10000 \left(1 + \frac{0.08}{12}\right)^{240} \\
&= \$26,000
\end{align*}
\]
"Annual Yield"

What is the annual yield of an account that earns 8% interest, compounded monthly?

Key question: What percent growth happens in one year?

\[100 \left(1 + \frac{0.08}{12}\right)^{12} = 108.29\ldots\]

So, annual yield is \(8.3\)% (about)

"Doubling Time"

What is the doubling time for an account that earns 6.5%, compounded weekly?

Key question: How long does it take for a starting amount to double?

\[
\frac{200}{100} = 100 \left(1 + \frac{i}{100}\right)^{2}\]

\[2 = (1 + \frac{0.065}{52})^{52\cdot t}\]

\[
\log(2) = \log \left(1 + \frac{0.065}{52}\right) \cdot 52\cdot t
\]

\[\log(2) = 52\cdot t \cdot \log(1 + \frac{0.065}{52})\]

\[
\frac{\log(2)}{52 \cdot \log(1 + \frac{0.065}{52})} = t
\]

Calculator: 10.67... about 10.7 years

What size deposit do you need to make now in order to end up with $10,000 at the end of 10 years? Assume you can get a 5% interest rate, compounded monthly.

Formula: \(FV = P \left(1 + \frac{i}{100}\right)^{n}\)

\[
10000 = P \left(1 + \frac{0.05}{12}\right)^{120}
\]

\[
\frac{10000}{\left(1 + \frac{0.05}{12}\right)^{120}} = P \approx \$6071.61
\]
Preparation for 5.3:

Geometric Series:

\[ a_1 + a_1r + a_1r^2 + a_1r^3 + \cdots + a_1r^{n-1} \]

Formula for the sum:

\[ \text{Sum} = \frac{a_1(1-r^n)}{1-r} \]

Current Formulas (Memorize these!):

- \( I = Prt \) (Interest by itself)
- \( FV = P(1+rt) \) (Simple interest)
- \( FV = P(1+r)^n \) (Compound interest)